HISTORIA MATHEMATICA **23** (1996), 437–446
ARTICLE NOS. 0041–0043

REVIEWS

Edited by CATHERINE GOLDSTEIN AND PAUL R. WOLFSON

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Development of Mathematics, 1900–1950. Edited by Jean-Paul Pier. Basel/Boston/Berlin (Birkhäuser-Verlag). 1994. 729 pp. DM 118. £ 43.

Reviewed by JEREMY GRAY

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Will we ever know the history of mathematics in the 20th century? The most likely answer is surely that we, at least, will not. Not because too much is too well hidden or too difficult to recover, as is the case with the political history of that violent century. Not because histories that are too Eurocentric are nowadays rightly dismissed as too narrow, thus enlarging the task impossibly. But simply because what is on the surface, written in a few well-known languages and published in accessible journals and books, is too much and too hard for us to master. Learning the history with all its imprecisions is harder, in some ways, than learning the mathematics (albeit easier in others), and we can wonder if we are asking the right questions. Before turning to the essays this book offers, consider its first 34 pages. This is a chronological list of 1000 important papers and books published between 1900 and 1950, compiled by asking some 50 experts. It cannot reasonably be supposed that any one person will have read these works and formed a fresh and accurate opinion of them, let alone of their much larger penumbra. Nor, of course,

0315-0860/96 $18.00

has the editor asked just one person to write the book. Instead we have 12 people who write about different aspects, and still much is left out. It could not be otherwise. Nor will the reader of this review fail to see that this single reviewer has his limitations.

The essays agree in a number of methodological ways. They present the history of mathematics as the growth of numerous branches of mathematics and so as the history of ideas. There are no quantitative methods here, no attempt to describe whole communities, almost no social history, and very little biography. There is very little attempt to show why the chosen subjects matter, or mattered in their own day. Why algebraic topology gets 115 pages of text and 6 of bibliography but partial differential equations only get 21 pages of text and 16 of bibliography is left unexplained. But we know the answer; it simply reflects the expertise available to Pier and his colleagues at that time faced with the impossible task of conveying the history of mathematics, 1900–1950. Even so, the choice of topics, with so much on mathematical logic (137 pages of text, 47 of bibliography), geometry subsumed by topology, and no algebra, makes one want to ask what belongs to mathematics, and what not?

As it happens, the essay on the history of topology is a gem. Jean Dieudonné was able to stand back from the details described in his much longer book [1] and give a remarkable account, with hints of motivation, helpful examples, and indications of personal and intellectual connections; the result is algebraic topology as a living subject, driven by the curiosity and the insights of numerous leading mathematicians (and not particularly constrained by the terminus of 1950 either). The omissions are part of the success of the story; the reader travels light, seeing worthwhile problems and their (sometimes) partial solutions.

Two shorter essays follow which can be read as offering methodological spice, even a hint of dissent. Doob's essay, "The Development of Rigor in Mathematical Probability, (1900–1950)," is a delight. The title already indicates that a sensible way through a vast amount of material has been found. Almost at once Doob writes: "Specific results are mentioned only in so far as they are important in the history of the logical development of mathematical probability." This is a good criterion for selection. Then he offers three famous opinions (by Planck, Poincaré, and Hermite) on progress in their subjects and follows it with three on the law of large numbers (that it is a theorem, a proposition, and a fact) and five on the definition of probability. Doob modestly deduces that this conflict of opinion requires one to separate mathematical probability from its real world applications, but it does much more: it establishes that there was a real debate about a substantial and difficult issue. Then he turns to measure theory, notes some of the work on Brownian motion, work of Borel, and Kolmogorov's memoir of 1933 (which was not immediately accepted). The author concludes by criticising the strange way in which many mathematicians try to hold measure theory and probability theory apart.

The next short essay, by Fichera, focuses on the evaluation of Volterra's work on functional analysis. He draws out its presuppositions and shows accordingly what it could do and where it was misleading, thus explaining its impact—ultimately negative—in Italy, because of Volterra's prestige. This vindicates the well-placed criticism of Dieudonné's account of the same material with which Fichera begins his essay: the distinction between doing history and writing an historical review. The latter exercise admits modern insights denied to the protagonists and invites historical misrepresentation. Reader—beware.

The other long essay, by Guillaume on mathematical logic, raises an awkward historical problem. No one disputes the profundity of the topic, but the field is deep and narrow, its relation to the rest of mathematics is difficult to elucidate and since the 1930s has become tenuous. Guillaume begins by invoking a number of the 19th-century sources of mathematical logic and then surveys a wide variety of issues in set theory, logic, and the foundations of mathematics. Despite the clarity of each paragraph, the overall effect is confusing. One is left wondering what the import of all this material was (in its day) and is (for the historian). In 137 pages of text and 47 bibliography, there has been some attempt at completeness, even though the author rightly admits at the very start that a completely faithful history would require several hundred pages. What, indeed, was the historical question at issue here? Most likely an attempt to say a bit about almost everything that seems to have lasted. Now it is a commonplace that Gödel's theorems put an end to Hilbert's programme to rigorise mathematics and with it the serious commitment of mathematicians to mathematical logic and the foundations of mathematics. The famous indifference of Bourbaki to these issues belongs here. But a commonplace need not be true. One might reasonably ask an historian to confront the question, and one might well ask Guillaume because his essay takes a sharp turn with Gödel's work. After the mid-1930s it becomes an account of specialists doing difficult technical work, innocently reinforcing our sense (if we are not logicians) of its remoteness. But although Gödel's theorems are said to have been an earthquake—and nine specific issues are listed as flowing from them as well as quite an amount of literature (good and not so good)—one misses any sense of what their historical significance was. If indeed the 19th-century programmes to make sense of mathematics all perished at this moment, then how and why? (I owe to a conversation with Ray Monk the realisation that the matter really is not simple.) Is the subsequent work as a result as dry, even irrelevant, as it seems? Better use of the growing historical literature might have helped to shape this essay.

The reader now enters the second half of the book: eight essays in just under 250 pages. There is a skill needed here by all of us, for the 30-page essay is what books and journals like. It helps if you have a topic of about the right size. Houzel's account of the prehistory of the Weil conjectures is one such topic. A small number of mathematicians progressively elucidated an area until one could formulate a series of rich questions which even as they were asked issued a profound challenge to the existing techniques (a rich mixture of algebraic number theory and algebraic geometry). Kahane's essay on Taylor series and Brownian motion addresses the issue of what mathematicians meant when they said something held in general; the question is a good one and the answer instructive. Mahwin's account of how questions in nonlinear ordinary differential equations led to the work of, amongst others, Leray and Schauder is rapid and dense, but it shares with Dieudonné's the feel that questions and problems led, in a natural way, to new methods and new problems. One picks up a sense of excitement.

The editor of the book, Jean-Paul Pier, contributed an essay on integration and measure (but not probability). It shares with Mahwin's the ability to connect things, to show this mathematician responding to that one. The emphasis is on the power of the techniques being developed, and it would have been interesting to see them do more; they were not, after all, put forward just for their own sake. The connection with functional analysis is left until the appearance of Schwartz, and that is also a pity, but this is the inevitable downside of a correct decision to pursue one aspect and make it intelligible.

The essays by Lichnerowicz, Nirenberg, Hayman, and Schwarz are less successful. The first of these is perhaps too short, with the result that the mathematics is too far away to be seen clearly, and one gets generalisations where precision was needed. Nirenberg's essay on partial differential equations is little more than a list of results (who first proved what, when). A whole book needs to be written on the topic, but in a limited space it would surely have been better to tell much less, with more spirit. Much the same can be said of Hayman's report on topics in complex analysis, which also says little new, but with less excuse. The paradox here is that while in its day Nevanlinna's theory was highly praised (by Hilbert and Weyl, no less), the topic of single variable complex function theory has ever since dwindled in esteem. This is not the type of historical development this genre of book can deal with easily. Schwarz's essay on the prime number theorem is equally factual, chronological, and unexciting.

The presence of Jean Dieudonné dominates the book. His is the first photograph to appear, and the first essay, but influence is felt in less visible ways: not perhaps in the choice of topics, but in the approach to history. This book is written in the dominant mode of history of mathematics, which emphasises the mathematical results. I have no criticisms of that mode, provided that one admits others elsewhere. What turns out to be curiously intangible is how the same approach can produce excitement in one essay and boredom in another. It may be partly what the reader (or reviewer) brings to the topic. It may be a literary skill. But sometimes the narrative mode is gripping: you want to read on, to know what happened next and why. Sometimes the result is facts, and one longs to bring them to life with questions. The story-telling skill of Dieudonné, the astute criticism of Fichera, and the dexterity of Doob are good examples for the historian to ponder, not least because they do not point in the same direction.

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Geschichte der Algebra. Eine Einführung. Edited by Erhard Scholz. Unter Mitarbeit von Kirsti Andersen *et al.* Lehrbücher und Monographien zur Didaktik der Mathematik Bd. 16. Wien/Zürich (B.I.-Wissenschafts Verlag Mannheim). 1990.

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It is an ambitious undertaking to write a satisfying history of algebra, especially if it is, at first glance, aimed at teachers and those who are interested in mathematics education. The present introduction is a collection of 15 contributions, which cover on the whole more than 500 pages. It appeared as volume 16 in a well-known, widespread series called "Lehrbücher und Monographien zur Didaktik der Mathematik" (eds. N. Knoche & H. Scheid).

Although the book is addressed primarily to teachers or students of mathematics (p. 1), one does not expect a mere history of elementary school algebra. Nevertheless, the present book sometimes even goes beyond the contents of the usual university-level algebra lecture. In this respect, the history of the theorems of Wedderburn in the theory of algebras (box, p. 408) or the theorem of Hilbert–Schmidt (box, p. 416) goes beyond an introductory history of algebra. On the other hand, a more thorough consideration of the early Egyptian, Chinese, Japanese, and Indian cultures, which would have been useful for pedagogical purposes, was not included.

The authors of the book are selected historians of mathematics, who are specialists in their field. This guarantees that the contributions offer a high degree of accuracy and reliability. This is a great advantage of the book!

Each of the 15 contributions is included in one of three parts, covering three chronological periods: ancient times and the medieval era; the Renaissance and following times (15th–18th century); and modern times (19th–20th century). The contributions cover at least 11 pages (Chapter 12) and at most 53 pages (Chapter 7).

Several summarized texts are highlighted by a so-called "box." In particular, the boxes on p. 19 (sexagesimal system), p. 48 (chronological table of ancient Greek mathematics), p. 312 (Gauss's theory of quadratic forms), and p. 423 (Public Key Code) provide the reader with a useful summary. It is worthwhile to examine the contents of the book in more detail.

Chapter 1 deals with algebraic thought in pre-Greek times under the title " 'Algebraische' Prozeduren in der vorgriechischen Mathematik." The Danish author Jens Høyrup—as becomes clear in his first paragraph "Subwissenschaftliche Traditionen" (1.1)—prefers to speak of "algebra" or protoalgebra instead of algebra (p. 38). His somewhat unusual interpretations of texts with geometrical connotations (p. 9) are relatively novel. He first discusses typical problems, mainly of Babylonian sources, dealing with equations of second and higher degree (1.2–1.6). On the other hand, he hardly touches upon the papyrus manuscripts of Egyptian times which would be of some interest in the teaching of mathematics in schools.

The editor concludes this part by hinting (p. 9) at the "traditional" "Vorgriechische Mathematik" by Kurt Vogel [4].

In Chapter 2, Ivor Bulmer-Thomas also provides a new interpretation, but of the origins of Greek algebraic thought. He does not adopt the traditional position that Greek implicit algebraic forms of thought are "geometrical algebra." Rather, he leaves the interpretation of the sources largely to the reader. In particular, he analyzes Book VI of Euclid's *Elements* (2.1), the Greek theory of proportion (2.3), and the complex cattle problem (considered by Archimedes) together with some problems of Heron (2.4).

The following three chapters, which complete part I, are carefully written in a very clear and intelligible manner. All by Jacques Sesiano, each chapter opens with a helpful summary that preceeds the detailed text. In Chapter 3, Sesiano discusses the "Arithmetica" of Diophantus, the only remaining Greek work on indefinite algebra. This collection of about 260 problems seems to be a "Gemisch aus Mathematik und Geschicklichkeit" (p. 94). Typical problems are taken in turn, such as quadratic equations and systems of linear equations.

Chapter 4 begins with a discussion of the significance of Chinese and Indian mathematics (4.1). It deals with the determination of square and cubic roots and systems of linear equations. The chapter itself concerns Arabic–Islamic algebra, its foundation and development. The Baghdad School, the exposition of recreational problems, the preservation of the Greek heritage by Arabic mathematicians, and the origin of the word "algebra" are of special interest. (In the classic Arabic manuscripts, the lack of symbols is noteworthy. Mathematics was expressed in words, i.e., "rhetorically.") Sesiano also presents the solutions of equations of second (4.3, 4.4) and third degree (4.4), performed by the Arabs in a geometrical way. The solutions still had to be positive, although irrational solutions were increasingly accepted. Gradually, algebra began to move away from geometric foundations. Among the practical applications, the "birds problem" is discussed.

As described in Chapter 5, above all with the reconquering of Spain, ancient ideas came to Europe through Arabic manuscripts. The contributions of Johannes of Hispalensis (12th century) in his book about commercial calculation are also of a certain educational interest. He solves problems first using a formula, then in a geometrical way, and finally by algebraic derivation (p. 133ff). Typical examples of problems by Leonardo of Pisa (5.3), which lead to systems of linear equations, are also presented. Through these, Leonardo came across negative solutions and the linear dependence of equations. Sesiano points out the first steps to the common solutions of equations of third and fourth degree developed in the 14th and 15th centuries (5.4) as well.

At the beginning of the second part of the book, Chapter 6 describes algebra in the Renaissance. In addition to the development of commercial arithmetic—which led to the profession of the independent mathematics teaching master and to the promotion of the Indian-Arabic style of writing numbers—the most important innovations of the Renaissance are seen in the contributions made in solving equations of the third or fourth degree. At the end of the 11th century, Omar al-Khayyam could solve all types of cubic equations in a geometrical way; the first publication of algebraic solutions goes back to Cardano (1545) (6.2). Kirsti Andersen describes the interesting history of that publication, while also considering Tartaglia's role in it. She examines these solutions in detail and points out that the irreducible case was the main stimulus for the introduction of complex numbers (p. 173). The solution of the fourth-degree equation can be traced back to Cardano's student Ferrari.

Chapter 7 moves to the work of Viète and Descartes and examines the evolution of the conceptual distinction between geometry and algebra. Algebra became a theory of equations, enlarged to abstract quantities, so it could be applied to both pure numerical and geometrical or physical problems. As she did in [3], Karin Reich analyzes Viète's role in this process of conceptual change, particularly his notion of n-dimensional quantities (7.2, 7.3). In 7.4, Henk Bos describes how Descartes, on the other hand, distanced himself from the dimensional interpretation. Later, quantities and relations were gradually replaced by mathematical symbols. Above all, according to Descartes, geometry was the skill of solving problems of geometrical constructions and not of proving geometrical theorems. Several examples in sections 7.5 to 7.9 present Descartes's new methods of translating problems into algebraic equations with the aid of special techniques. The question of algebraic structures and their development is profoundly connected with the different approaches of Descartes and Viète, and, as is well-known, the ideas of Descartes eventually won out. The following chapters explore the effects of algebraic methods on different mathematical fields up through the beginning of the 19th century.

In Chapter 8, Ivo Schneider studies the role of algebraic thought in the early theory of probabilities. He argues that, up to the work of Christiaan Huygens (1629–1695), algebra was an important mathematical tool (8.2) (cf. [1]). Then, beginning with Abraham de Moivre (1667–1754), algebra was increasingly displaced in probability theory by the methods of analysis. With the work of de Moivre, the theory of probabilities became a part of mathematics proper.

Catherine Goldstein describes 17th- and 18th-century number theory in Chapter 9 from an algebraic point of view. She investigates particular contributions by Fermat (9.1), Euler (9.3), and Lagrange (9.4) and traces the historical path from the study of equations and expansions in series to a structural algebra increasingly concerned with binary forms and questions of classification.

In the following chapter, Jeremy J. Gray describes the relation between algebra and geometry in the 18th and 19th centuries—especially the role of the new projective geometry which was created by Poncelet (10.4), Plücker, and Möbius (10.5), among others. The concept of projective transformation turned out to be the decisive concept in the classification of algebraic curves. The question about complex points of curves led to the fundamental theorem of algebra which was proved in several ways by Gauss beginning in 1799.

The third part of the book concerns the development of modern algebra. In Chapter 11, Gray begins with a review of the genesis of complex numbers. Beginning with Gauss and Hamilton, mathematicians increasingly withdrew from a metaphysical justification until, by the end of the 19th century, mathematical existence was constituted by a system of noncontradictory axioms. The author then examines in detail the origin of the concepts of group, ring, and ideal (11.3–11.6).

Gray concludes this part with a summary (Chapter 12) of the development of the European university system. In the late 18th and 19th centuries, universities were changed from educational academies to institutions in which education and research were appreciated in an equivalent way.

In Chapter 13, Erhard Scholz studies the history of linear algebra, especially the origins of the concept of vector space in the work of Hermann Grassmann (1844), who drew attention to several applications such as systems of linear equations, geometry, physics, and crystallography. Hamilton's quaternions (13.4), other hypercomplex systems (13.8), the diagonalization of linear systems (13.6), and applications in physics and electrodynamics (13.9) are all remarkable elements of what became unified into linear alegbra in the 20th century.

Scholz describes the origins of Galois theory in Chapter 14, focusing on the definition of the algebraic structures of field extensions and automorphism groups. While the contributions by Hudde and Lagrange (14.1) and Ruffini, Gauss, and Abel (14.2) belong to the prehistory of the subject, the work of Galois (14.3–14.5) forms its nucleus, and Scholz presents Galois's often obscure ideas in an intelligible way. He also discusses the elaboration of Galois's work by Liouville, Jordan, Betti, Kronecker, and Dedekind (14.6–14.7). In (14.8), the origin of the concept of field is described.

Chapter 15 presents a résumé of the development of abstract algebra in the first two thirds of the 20th century. It traces the concept of axiomatization at least back to Hilbert's *Foundations of Geometry* [2] of 1899 (15.1); examines the influence of Emmy Noether, the "mother of modern algebra" and a leading figure in Hilbert's Göttingen school (15.2); explores the consequences of Hilbert's program of formalization (15.3); and looks at the origins both of functional analysis (15.4) and Bourbaki's "Strukturalgebra" (15.5). The final impression is one of the important roles algebra has played in 20th-century mathematics.

The numerous original texts in this book are of special value. The facsimile reproductions of old texts, e.g., a Provençal manuscript (p. 143), Bombelli (p. 180), Cardano (p. 169), and the theorem of square roots by Viète (p. 197), are also of particular interest.

Besides this, the two significant and detailed lists of literature and sources, covering more than 30 pages, should be mentioned. If one looks for a certain reference, it is advisible to have both lists in mind. Some references in the text have been omitted from the lists, for instance, "Reichardt 1976" on p. 402 (15.1) or "Beaulieu 1989" on p. 461. As the editor indicates in the Introduction, it was not always possible to refer to German literature. This occasional disadvantage would be advantageous to the English-speaking reader because of the international team of authors.

The book is recommended not only for teachers who are interested in history, but also for a wide range of mathematically informed readers. Through the connection of "discovery–learning–teaching," the history of mathematics is profoundly interlinked with mathematics education. For the reader interested in the history of mathematics from a pedagogical point of view, however, it is not easy to isolate pedagogical illustrations in this book.

As mentioned in the introduction, the texts sometimes vary in their stylistic form. Nevertheless, it is obvious that the contents of the chapters have been harmonized for there are numerous cross-references to other chapters as well as annotations. The illustrating figures normally occur at the beginning of a new page, so text and figures sometimes become separated. Some figures need more attention. For example, in the description of the normal method by Descartes (7.8), in the text, the intersection point of the curve and the circle (p. 219 and p. 223 middle) is labeled "P," whereas in the corresponding figure and in the box, the center of the circle (p. 223 above) is also labeled "P."

Since the descriptions of figures and equations are not always consistent, it pays to orient oneself within a given chapter. For example, in Chapter 2 equations are, at one point, referred to as 2.1 to 2.9, and, at another, as (1) to (9). Proper names are sometimes emphasized using capitals or italics. The name index probably only includes just over half of all names; perhaps it could be completed in a future edition.

To sum up, a main attraction of this work lies in its stylistic differences. Although the book will not make for easy reading for everyone (the first two chapters are especially ambitious), the persistent reader will be rewarded by profound and stimulating insights into current research on the history of algebra. The authors and the editor should be applauded for all of their hard work in producing this text. The few criticisms registered above should in no way detract from a real appreciation of this work.

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Anaritus' Commentary on Euclid. The Latin Translation, I–IV. Edited by P. M. J. E. Tummers. Artistarium, Supplementa, IX. Nijmegen (Ingenium Publishers). 1994. 187 pp.

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Abu l-ᶜAbbās al-Faḍl ibn Ḥātim an-Nairīzī († 922) est connu dans l'Occident médiéval, sous le nom de Anaritius (ou Anarizus), pour son commentaire aux

Éléments d'Euclide, traduit en latin par Gérard de Crémone au milieu du XII^{è} siècle. Les commentaires aux Livres I à VI et aux premières Définitions du Livre VII sont conservés en arabe, accompagnant une traduction remaniée des *Éléments*. En latin ne sont conservés que les commentaires,[1] mais pour les Livres I à X.

Ce texte est important pour plusieurs raisons. Comme de nombreux commentaires arabes il contient d'importantes discussions sur les "lieux" les plus célèbres des *Éléments*:

—les demandes et axiomes (en particulier la 5^{è} Demande, ou postulat des parallèles; an-Nairīzī transmet la "preuve" d'Aganis);

—les notions de rapport et de proportionnalité du Livre V (an-Nairīzī mentionne l'approche anthyphérétique (algorithme d'Euclide) que les Latins, malgré la traduction de Gérard, ne reprendront apparemment pas); et

—la théorie des irrationnelles du Livre X.

Alors que de nombreux textes arabes sont consacrés à l'un de ces points seulement, le commentaire d'an-Nairīzī couvrait vraisemblablement la totalité des *Éléments*; cette caractéristique mérite d'être relevée. C'est sans doute ce qui fait dire à Tummers (p. ix) que le texte d'an-Nairīzī est le seul commentaire arabe aux *Éléments* d'Euclide à avoir été traduit en latin durant le Moyen-Âge. Il faut sans doute comprendre commentaire "complet"; car, comme l'auteur le signale lui-même (note 36, pp. 145–146), le commentaire attribué à Muḥammad ibn ^{c}Abd al-Bāqī al-Baghdādī sur le Livre X a également été rendu en latin, sans doute aussi par Gérard de Crémone, à la suite de celui d'an-Nairīzī. On pourrait même leur adjoindre la traduction du début du commentaire au Livre X attribué à Pappus, et dont l'original grec était perdu. Mais celui d'an-Nairīzī était le plus complet et sa traduction latine a été connue de Roger Bacon, Albert le Grand, et très certainement de Campanus.

Enfin autre intérêt, cette fois du point de vue de l'histoire des mathématiques grecques, an-Nairīzī cite les commentaires composés par Simplicius et par Héron[2] dont les originaux grecs ne nous sont pas parvenus.

L'intérêt épistémologique et historique de ce texte est donc évident. Jusqu'ici on disposait de l'édition de Maximilien Curtze[3] établie sur la base d'un unique manuscrit de qualité médiocre selon Curtze lui-même. Tummers nous donne une nouvelle édition des commentaires aux Livres I–IV prenant en compte trois nouveaux manuscrits. Il promet celle des commentaires aux Livres V à X pour un prochain volume. En plus de l'édition critique du texte latin le lecteur trouvera en Appendice les références des emprunts faits par Roger Bacon à Anaritius. Un index des termes latins utilisés et une bibliographie complètent le tout. On regrettera simplement qu'elle s'arrête à 1984.

[1] Au demeurant Gérard avait aussi produit une traduction latine complète du traité euclidien.

[2] Le plus ancien qui nous soit connu en admettant pour Héron la date proposée par Neugebauer: le milieu du premier siècle de notre ère.

[3] *Anaritii in decem libros priores Elementorum Euclidis Commentarii*, in *Euclidis Opera omnia*, Supplementum, ed. I. L. Heiberg & H. Menge, Leipzig, in aed. B. G. Teubner, IX, 1899.